strong interaction effects, such as reflected shocks and rarefaction waves.

Other experimental investigations, mainly quantitative, are necessary, however, to prove that the observed reflected fronts are plane shock fronts and to determine all flow parameters. In addition, relaxation effects behind shock waves in plasmas can be studied.

Only then can the experimental results be reasonably compared with the theory. But the steady state cannot be achieved, as the results show. Therefore the transient part of the theory of JOHSON [2] must be taken as a basis. However, the assumption made in this theory that the current density can be expressed in terms of $\sigma \cdot V \cdot B$ is not valid. Furthermore, ionization effects have to be taken into account. More experimental and theoretical investigations are therefore required.

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Higher Harmonics Generation in Weakly Ionized Plasmas in Time-Varying Crossed Fields

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The linearized Boltzmann equation of kinetic theory for the electrons of a weakly ionized plasma under the action of an alternating electric field $\boldsymbol{E} = \cos \omega t \{E_x, E_y, E_z\}$ and a circularly polarized magnetic field $\boldsymbol{B}^{\mathrm{R}} = B \{\cos \omega_B t, \sin \omega_B t, 0\}$ is solved. The direction-dependent part $\boldsymbol{f}^1(\boldsymbol{r}, \boldsymbol{v}, t)$ of the distribution function contains especially higher harmonics with frequencies $2\,\omega_B$, $\omega \pm \omega_B$, $\omega \pm 2\,\omega_B$. These overtone frequencies are generated — unlike the higher harmonics obtained by Margenau and Hartman — by a cross drift mechanism and are present even if the isotropic part f^0 of the electron distribution function is assumed to be time-independent. The physical meaning of the various contributions to the distribution function $\boldsymbol{f}^1$ is discussed in detail.

1. Introduction

The influence of crossed electric and magnetic external fields on plasmas has been studied for several reasons and with different methods. For example, it is interesting to learn at which electric field strengths the breakdown in a high-frequency low pressure gas discharge starts. Furthermore, it is important to study the plasma particle motion in crossed field geometries in order to get information about a possible particle acceleration. This is of interest in cosmic ray physics, electron and ion optics etc. Finally a large amount of effort has been devoted to the examination of appropriate crossed field configurations for an effective heating and a stable confinement of high temperature plasmas. Often it is sufficient and less expensive to examine these problems in suitable model plasmas, e. g. using the weakly ionized regions of low pressure discharges [1].

Many investigations have been performed in which the crossed fields are assumed to be static. With respect to the applications mentioned it is also important to study the plasma behaviour if the ex-

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ternal electric field is time-dependent (being mostly an alternating field in practice) and the magnetic field is constant [2–4], or vice versa [5–7]. Finally new phenomena are expected to appear if both the external electric field and the magnetic field are time-varying and independently established [8].

An approximative theoretical treatment of such problems consists in using the single particle description [9, 10]. In a similar way the equations of motion of magneto-hydrodynamics can be used [6]. Finally it is possible to resort to the powerful methods of kinetic theory or statistical mechanics [11–15]. In this paper a weakly ionized plasma is investigated using Boltzmann's kinetic equation. The plasma is assumed to be under the action of an alternating electric field, a circularly polarized magnetic field, and additional constant fields.

2. Basis Equations

Let us assume the heavy particles of a weakly ionized plasma to have a Maxwellian distribution function. For the electron distribution function the Lorentz ansatz is made

$$f(\mathbf{r}, v, t) = f^0(\mathbf{r}, v, t) + \frac{\mathbf{v}}{v} \cdot \mathbf{f}^1(\mathbf{r}, v, t), \quad |\mathbf{v}| = v. \tag{1}$$

The coupled system of differential equations for the calculation of the parts f^0 and $\mathbf{f}^1$ is obtained from the Boltzmann equation [16]:

$$\frac{\partial f^0}{\partial t} + \frac{v}{3} \operatorname{div}_r \mathbf{f}^1 + \frac{1}{3 v^2} \frac{\partial}{\partial v} \left(v^2 [\mathbf{E} + \mathbf{E}_\mathrm{i}] \cdot \mathbf{f}^1\right) = \left.\frac{\delta f^0}{\delta t}\right|_{\mathrm{e-n}} \tag{2}$$

where

$$\left.\frac{\delta f^0}{\delta t}\right|_{\mathrm{e-n}} = \frac{1}{v^2} \frac{m}{M} \frac{\partial}{\partial v} \left(v^3 \nu f^0 + \frac{k T}{m} v^2 \nu \frac{\partial f^0}{\partial v}\right),$$

$$\frac{\partial \mathbf{f}^1}{\partial t} + v \operatorname{grad}_r f^0 + \frac{q}{m} (\mathbf{E} + \mathbf{E}_\mathrm{i}) \frac{\partial f^0}{\partial v} + \frac{q}{m} [\mathbf{B} \times \mathbf{f}^1] = -\nu \mathbf{f}^1. \tag{3}$$

[$\nu(v)$: electron-neutral collision frequency, q/m: electronic charge-to-mass ratio, M: mass of a neutral particle, T: gas temperature, $\mathbf{E}$: electric field, $\mathbf{B}$: magnetic field.] The following field configuration is considered:

(a) homogeneous alternating electric field $\mathbf{E}(t)$ with

$$\boldsymbol{\alpha} = \boldsymbol{\alpha}(t) = (q/m)\, \mathbf{E}(t) = (q/m) \cos \omega t \{E_x, E_y, E_z\}, \tag{4}$$

(b) homogeneous time-independent electric field $\mathbf{E}_0$ with

$$\boldsymbol{\alpha}_0 = \{\alpha_{0x}, \alpha_{0y}, \alpha_{0z}\} = (q/m) \{E_{0x}, E_{0y}, E_{0z}\} \tag{5}$$

(c) homogeneous circularly polarized magnetic field $\mathbf{B}^\mathrm{R}(t)$ with

$$\boldsymbol{\omega}^\mathrm{R} = (q/m)\, \mathbf{B}^\mathrm{R} = \omega_\mathrm{C} \{\cos \omega_\mathrm{B} t, \sin \omega_\mathrm{B} t, 0\}, \quad \omega_\mathrm{C} = (q/m)\, B. \tag{6}$$

(d) electric field $\mathbf{E}_\mathrm{i}^\mathrm{R}$ induced by the time-varying magnetic field $\mathbf{B}^\mathrm{R}(t)$ with

$$\boldsymbol{\alpha}_\mathrm{i}^\mathrm{R} = \boldsymbol{\alpha}_\mathrm{i}^\mathrm{R}(\mathbf{r}, t) = (q/m)\, \mathbf{E}_\mathrm{i}^\mathrm{R}(\mathbf{r}, t) = \omega_\mathrm{C}\, \omega_\mathrm{B} \{0, 0, x \cos \omega_\mathrm{B} t + y \sin \omega_\mathrm{B} t\}. \tag{7}$$

(e) homogeneous direction-constant time-independent magnetic field B_0 with

$$\boldsymbol{\omega}_0 = \{0, 0, \omega_0\} = (q/m) \{0, 0, B_0\}. \tag{8}$$

Making use of the abbreviations

$$w(\mathbf{r}, v, t) = f_x^1(\mathbf{r}, v, t) + i f_y^1(\mathbf{r}, v, t), \qquad G_w = v \left(\frac{\partial}{\partial x} + i \frac{\partial}{\partial y}\right),$$
$$A_w = (q/m)(E_x + i E_y) = \alpha_x + i \alpha_y, \qquad B^\varrho(t) = -i \omega_\mathrm{C} \exp\{i \omega_\mathrm{B} t\},$$
$$A_{w\mathrm{i}}^\mathrm{R} = (q/m)(E_{\mathrm{i}x}^\mathrm{R} + i E_{\mathrm{i}y}^\mathrm{R}) = \alpha_{\mathrm{i}x}^\mathrm{R} + i \alpha_{\mathrm{i}y}^\mathrm{R}, \qquad A_{0w} = \alpha_{0x} + i \alpha_{0y} \tag{9}$$

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one can compose the first and second component equation of (3) to:

$$\frac{\partial w}{\partial t} + i(\omega_0 - i\nu)\, w = -\left(G_w f^0 + [A_{0w} + A_w \cos\omega t + A^{\mathrm{R}}_{\mathrm{i}w}] \frac{\partial f^0}{\partial v} - B^{\varrho}(t)\, f^1_z\right) = S^{\mathrm{R}}_w(\mathbf{r}, v, t). \tag{10}$$

In a similar fashion the z component of Eq. (3) can be written as

$$\frac{\partial f^1_z}{\partial t} + \nu f^1_z = -\left(G_z f^0 + [A_{0z} + A_z \cos\omega t + A^{\mathrm{R}}_{\mathrm{i}z}] \frac{\partial f_0}{\partial v} - \mathrm{Re}\{\overline{B^{\varrho}}\, w\}\right) = S^{\mathrm{R}}_z(\mathbf{r}, v, t) \tag{11}$$

where

$$G_z = v\frac{\partial}{\partial z}, \qquad A_{0z} = \alpha_{0z}, \qquad A_z = \alpha_z, \qquad A^{\mathrm{R}}_{\mathrm{i}z} = \alpha^{\mathrm{R}}_{\mathrm{i}z}. \tag{12}$$

Let us consider now only the solutions ${}^{\mathrm{inh}}w$ and ${}^{\mathrm{inh}}f^1_z$ of the inhomogeneous differential equations of (10) and (11), respectively [15]:

$$w(f^0, f^1_z) = {}^{\mathrm{inh}}w(\mathbf{r}, v, t) = \left\{\int_0^t \mathrm{d}\tau\, S^{\mathrm{R}}_w(\mathbf{r}, v, \tau; f^0, f^1_z) \exp[(\nu + i\omega_0)\tau]\right\} \exp\{-(\nu + i\omega_0)\, t\}, \tag{13}$$

$$f^1_z(f^0, w) = {}^{\mathrm{inh}}f^1_z(\mathbf{r}, v, t) = \left\{\int_0^t \mathrm{d}\tau\, S^{\mathrm{R}}_z(\mathbf{r}, v, \tau; f^0, w) \exp[\nu\tau]\right\} \exp\{-\nu t\}. \tag{14}$$

Firstly it must be pointed out that these "solutions" are formal ones with respect to the unknown isotropic part f^0. The difficulty that the quantity f^1_z still contains w (and vice versa) can be overcome if one substitutes $w(f^0, f^1_z)$ into the solution $f^1_z(f^0, w)$. The resulting integrodifferential equation for $f^1_z(r, v, t; f^0)$ has the form

$$\frac{\partial f^1_z}{\partial t} + \nu f^1_z + \omega_c^2\, \mathrm{Re}\left\{\int_0^t \mathrm{d}t'\, f^1_z \exp([\nu + i(\omega_0 + \omega_{\mathrm{B}})]\,(t' - t))\right\} = I(\mathbf{r}, v, t; f^0) \tag{15}$$

where

$$I(\mathbf{r}, v, t; f^0) = -G_z f^0 - (A_{0z} + A_z \cos\omega t + A^{\mathrm{R}}_{\mathrm{i}z}) \frac{\partial f^0}{\partial v}$$
$$-\omega_{\mathrm{C}}\, \mathrm{Re}\left\{i\left[\int_0^t \mathrm{d}t' \left(G_w f^0 + A_{0w}\frac{\partial f^0}{\partial v} + A_w \cos\omega t' \frac{\partial f^0}{\partial v}\right) \exp([\nu + i\omega_0]\, t')\right\} \exp\{-[\nu + i(\omega_0 + \omega_{\mathrm{B}})]\, t\}. \tag{16}$$

By repeatedly differentiating Eq. (16) with respect to time and substituting terms from higher time derivatives by lower ones one can get a linear inhomogeneous third-order differential equation for $f^1_z(\mathbf{r}, v, t; f^0)$:

$$\dddot{f}^1_z + 3\nu \ddot{f}^1_z + (3\nu^2 + \omega_c^2 + [\omega_0 + \omega_{\mathrm{B}}]^2)\, \dot{f}^1_z + \nu(\nu^2 + \omega_c^2 + [\omega_0 + \omega_{\mathrm{B}}]^2)\, f^1_z = \ddot{I} + 2\nu \dot{I} + (\nu^2 + [\omega_0 + \omega_{\mathrm{B}}]^2)\, I. \tag{17}$$

With help of the Hurwitz criterion it can be easily proved that the solutions ${}^{\mathrm{h}}_{i}f^1_z\,(i = 1, 2, 3)$ of the homogeneous differential equation of (17) asymptotically disappear for $t \to \infty$. Therefore the task consists in calculating the quantity ${}^{\mathrm{inh}}f^1_z(\mathbf{r}, v, t; f^0)$. Then one can find ${}^{\mathrm{inh}}w(\mathbf{r}, v, t; f^0)$, too, by substituting ${}^{\mathrm{inh}}f^1_z(\mathbf{r}, v, t; f^0)$ into Eq. (13). The combination of the quantities ${}^{\mathrm{inh}}w$ and ${}^{\mathrm{inh}}f^1_z$ with Eq. (2) then gives an equation for f^0 alone. The solution of this equation and the final calculation of the parts f^0 and $\mathbf{f}^1$ which are not necessary for the following considerations can be found in a further paper [17].

3. Calculation of the Distribution Function Part ${}^{\mathrm{inh}}f^1_z(\mathbf{r}, v, t; f^0)$

Now the isotropic part f^0 is assumed to be time-*independent*. This is tantamount to the fact that the relaxation time of the electronic energy is much greater than the momentum relaxation time.

[17] W. STILLER and G. VOJTA, Z. Naturforsch. **24 a** [1969], to be published.

In order to alleviate the solution procedure of Eq. (17) the term $I(\mathbf{r}, v, t; f^0)$ is transformed in an appropriate manner. First one integrates over the time variable t'. Only the solution ${}^{\text{inh}}f_z^1(\mathbf{r}, v, t; f^0)$ for the stationary state is interesting; that means that terms with a factor $\exp[-\nu t]$ appearing in $I(\mathbf{r}, v, t; f^0)$ can be neglected. Equation (17) transforms into

$$\dddot{f}_z^1 + 3\nu \ddot{f}_z^1 + (3\nu^2 + \omega_C^2 + [\omega_0 + \omega_B]^2)\, \dot{f}_z^1 + \nu(\nu^2 + \omega_C^2 + [\omega_0 + \omega_B]^2)\, f_z^1$$
$$= -\left(\alpha_{0z} \frac{\partial f^0}{\partial v} + G_z f^0\right) - \alpha_z \cos \omega t \frac{\partial f^0}{\partial v} - \omega_C \operatorname{Re}\left\{ i\left[A_{0w} \frac{\partial f^0}{\partial v} + G_w f^0\right] \frac{1}{\nu + i\omega_0} \exp\{-i\omega_B t\}\right\} \tag{18}$$
$$- \omega_C \frac{\partial f^0}{\partial v} \operatorname{Re}\left\{ i A_w \frac{(\nu + i\omega_0)\cos\omega t + \omega \sin \omega t}{(\nu + i\omega_0)^2 + \omega^2} \exp\{-i\omega_B t\}\right\} - \omega_C\, \omega_B\, (x \cos\omega_B t + y \sin \omega_B t) \frac{\partial f^0}{\partial v}\,.$$

Now for ${}^{\text{inh}}f_z^1(\mathbf{r}, v, t; f^0)$ the solution ansatz is made:

$$\begin{aligned} f_z^1 = {}^{\text{inh}}f_z^1(\mathbf{r}, v, t; f^0) = {}^0f_z^1(\mathbf{r}, v) &+ c_1(\mathbf{r}, v) \sin\omega t && + c_2(\mathbf{r}, v)\cos\omega t \\ &+ d_1(\mathbf{r}, v)\sin\omega_B t && + d_2(\mathbf{r}, v) \cos \omega_B t \\ &+ e_1(\mathbf{r}, v) \sin\{(\omega + \omega_B)\, t\} && + e_2(\mathbf{r}, v)\cos\{(\omega + \omega_B)\, t\} \\ &+ g_1(\mathbf{r}, v)\sin\{(\omega - \omega_B)\, t\} && + g_2(\mathbf{r}, v) \cos\{(\omega - \omega_B)\, t\}\,. \end{aligned} \tag{19}$$

Substituting this expression into (18) and equating the coefficients, the unknown quantities ${}^0f_z^1$, c_i, e_i, d_i, and g_i $(i = 1, 2)$ can be calculated containing in every case the isotropic part $f^0(\mathbf{r}, v)$. Using the abbreviations

$$\begin{aligned} f_z^1 &= \sum_{j=1}^{5} \langle Z_j \rangle, \\ \langle Z_1\rangle &\equiv \langle Z\,1\rangle = {}^0f_z^1, \\ \langle Z_2\rangle &\equiv \langle Z\,2\rangle = c_1 \sin\omega t + c_2 \cos\omega t, \\ \langle Z_3\rangle &\equiv \langle Z\,3\rangle = (\delta_1 + \Delta_1)\sin\omega_B t + (\delta_2 + \Delta_2)\cos\omega_B t = d_1 \sin\omega_B t + d_2 \cos\omega_B t, \\ \langle Z_4\rangle &\equiv \langle Z\,4\rangle = e_1 \sin\{(\omega + \omega_B)\, t\} + e_2 \cos\{(\omega + \omega_B)\, t\}, \\ \langle Z_5\rangle &\equiv \langle Z\,5\rangle = g_1 \sin\{(\omega - \omega_B)\, t\} + g_2 \cos\{(\omega - \omega_B)\, t\} \end{aligned} \tag{20}$$

and

$$\begin{aligned} \mathbf{U}_0 &= \{U_{0x}, U_{0y}, U_{0z}\} = \left\{\alpha_{0x} \frac{\partial f^0}{\partial v} + v \frac{\partial f^0}{\partial x},\ \alpha_{0y} \frac{\partial f^0}{\partial v} + v \frac{\partial f^0}{\partial y},\ \alpha_{0z} \frac{\partial f^0}{\partial v} + v\frac{\partial f^0}{\partial z}\right\}, \\ \mathbf{U} &= \{U_x, U_y, U_z\} = \left\{\alpha_x \frac{\partial f^0}{\partial v},\ \alpha_y \frac{\partial f^0}{\partial v},\ \alpha_z \frac{\partial f^0}{\partial v}\right\} \cos\omega t \end{aligned} \tag{21}$$

the solution ${}^{\text{inh}}f_z^1(\mathbf{r}, v, t; f^0)$ can be given in terms of Table 1. The decomposition of the coefficient d_1 (d_2) into additive parts δ_1 and Δ_1 $(\delta_2$ and $\Delta_2)$ shall emphasize the influence of the induced electric field denoted by Δ_1 and Δ_2.

The physical meaning of the terms included in ${}^{\text{inh}}f_z^1(\mathbf{r}, v, t; f^0)$ will be discussed in Section 5.

4. Calculation of the Distribution Function Part ${}^{\text{inh}}w(\mathbf{r}, v, t; f^0)$

The solution ${}^{\text{inh}}f_z^1(\mathbf{r}, v, t; f^0)$ can be written for short in the form

$${}^{\text{inh}}f_z^1 = {}^0f_z^1 + \operatorname{Re}\{-i[c\, e^{i\omega t} + d\, e^{i\omega_B t} + \varepsilon\, e^{i(\omega + \omega_B)t} + g\, e^{i(\omega - \omega_B)t}]\} \tag{22}$$

with $\quad c = c_1 + i c_2, \quad d = d_1 + i d_2, \quad \varepsilon = e_1 + i e_2, \quad g = g_1 + i g_2\,.$

Now (22) is substituted into (13) taking into consideration the meaning of S_w^R according to Eq. (10):

$$\begin{aligned} {}^{\text{inh}}w(\mathbf{r}, v, t; f^0) = &\left\{-\int_0^t \mathrm{d}\tau \left[A_{0w} \frac{\partial f^0}{\partial v} + G_w f^0\right] \exp\{(\nu + i\omega_0)\,\tau\}\right\} \exp\{-(\nu + i\omega_0)\, t\} \\ &- \left\{A_w \int_0^t \mathrm{d}\tau \cos\omega\tau \frac{\partial f^0}{\partial v} \exp\{(\nu + i\omega_0)\,\tau\}\right\} \exp\{-(\nu + i\omega_0)\, t\} \\ &+ \left\{ i\,\omega_C \int_0^t \mathrm{d}\tau ({}^0f_z^1 + \operatorname{Re}\{-i[c\, e^{i\omega\tau} + d\, e^{i\omega_B\tau} + \varepsilon\, e^{i(\omega + \omega_B)\tau} + g\, e^{i(\omega - \omega_B)\tau}]\})\right\}. \end{aligned} \tag{23}$$

For ⟨Z 1⟩	${}^{0}f_z^1(\boldsymbol{r}, v) = \dfrac{-(\nu^2+(\omega_0+\omega_B)^2)}{\nu(\nu^2+\omega\;\;+(\omega_0+\omega_B)^2)} U_{0z}$	
For ⟨Z 2⟩	$c_1(\boldsymbol{r}, v) = \dfrac{{}^{\omega}a_{11}\,{}^{\omega}b_{11} + {}^{\omega}a_{12}\,{}^{\omega}b_{12}}{({}^{\omega}a_{11})^2 + ({}^{\omega}a_{12})^2} \alpha_z \dfrac{\partial f^0}{\partial v}$	$c_2(\boldsymbol{r}, v) = \dfrac{{}^{\omega}a_{11}\,{}^{\omega}b_{12} - {}^{\omega}a_{12}\,{}^{\omega}b_{11}}{({}^{\omega}a_{11})^2 + ({}^{\omega}a_{12})^2} \alpha_z \dfrac{\partial f^0}{\partial v}$
For ⟨Z 3⟩	$\delta_1(\boldsymbol{r}, v) = \left[\dfrac{{}^{\omega_B}a_{11}\,{}^{\omega_B}b_{11} + {}^{\omega_B}a_{12}\,{}^{\omega_B}b_{12}}{({}^{\omega_B}a_{11})^2 + ({}^{\omega_B}a_{12})^2} U_{0x} + \dfrac{{}^{\omega_B}a_{11}\,{}^{\omega_B}b_{12} - {}^{\omega_B}a_{12}\,{}^{\omega_B}b_{11}}{({}^{\omega_B}a_{11})^2 + ({}^{\omega_B}a_{12})^2} U_{0y}\right]$	$\delta_2(\boldsymbol{r}, v) = \left[\dfrac{{}^{\omega_B}a_{11}\,{}^{\omega_B}b_{12} - {}^{\omega_B}a_{12}\,{}^{\omega_B}b_{11}}{({}^{\omega_B}a_{11})^2 + ({}^{\omega_B}a_{12})^2} U_{0x} - \dfrac{{}^{\omega_B}a_{11}\,{}^{\omega_B}b_{11} + {}^{\omega_B}a_{12}\,{}^{\omega_B}b_{12}}{({}^{\omega_B}a_{11})^2 + ({}^{\omega_B}a_{12})^2} U_{0y}\right]$
	$\Delta_1(\boldsymbol{r}, v) = \left[\dfrac{{}^{\omega_B}a_{11}\,{}^{\omega_B}b_{13} + {}^{\omega_B}a_{12}\,{}^{\omega_B}b_{14}}{({}^{\omega_B}a_{11})^2 + ({}^{\omega_B}a_{12})^2} x \dfrac{\partial f^0}{\partial v} + \dfrac{{}^{\omega_B}a_{11}\,{}^{\omega_B}b_{14} - {}^{\omega_B}a_{12}\,{}^{\omega_B}b_{13}}{({}^{\omega_B}a_{11})^2 + ({}^{\omega_B}a_{12})^2} y \dfrac{\partial f^0}{\partial v}\right]$	$\Delta_2(\boldsymbol{r}, v) = \left[\dfrac{{}^{\omega_B}a_{11}\,{}^{\omega_B}b_{14} - {}^{\omega_B}a_{12}\,{}^{\omega_B}b_{13}}{({}^{\omega_B}a_{11})^2 + ({}^{\omega_B}a_{12})^2} x \dfrac{\partial f^0}{\partial v} - \dfrac{{}^{\omega_B}a_{11}\,{}^{\omega_B}b_{13} + {}^{\omega_B}a_{12}\,{}^{\omega_B}b_{14}}{({}^{\omega_B}a_{11})^2 + ({}^{\omega_B}a_{12})^2} y \dfrac{\partial f^0}{\partial v}\right]$
For ⟨Z 4⟩	$e_1(\boldsymbol{r}, v) = \left[\dfrac{{}^{+}a_{11}\,{}^{+}b_{11} + {}^{+}a_{12}\,{}^{+}b_{12}}{({}^{+}a_{11})^2 + ({}^{+}a_{12})^2} \alpha_x \dfrac{\partial f^0}{\partial v} + \dfrac{{}^{+}a_{11}\,{}^{+}b_{12} - {}^{+}a_{12}\,{}^{+}b_{11}}{({}^{+}a_{11})^2 + ({}^{+}a_{12})^2} \alpha_y \dfrac{\partial f^0}{\partial v}\right]$	$e_2(\boldsymbol{r}, v) = \left[\dfrac{{}^{+}a_{11}\,{}^{+}b_{12} - {}^{+}a_{12}\,{}^{+}b_{11}}{({}^{+}a_{11})^2 + ({}^{+}a_{12})^2} \alpha_x \dfrac{\partial f^0}{\partial v} - \dfrac{{}^{+}a_{11}\,{}^{+}b_{11} + {}^{+}a_{12}\,{}^{+}b_{12}}{({}^{+}a_{11})^2 + ({}^{+}a_{12})^2} \alpha_y \dfrac{\partial f^0}{\partial v}\right]$
For ⟨Z 5⟩	$g_1(\boldsymbol{r}, v) = -\left[\dfrac{{}^{-}a_{11}\,{}^{-}b_{11} + {}^{-}a_{12}\,{}^{-}b_{12}}{({}^{-}a_{11})^2 + ({}^{-}a_{12})^2} \alpha_x \dfrac{\partial f^0}{\partial v} + \dfrac{{}^{-}a_{11}\,{}^{-}b_{12} - {}^{-}a_{12}\,{}^{-}b_{11}}{({}^{-}a_{11})^2 + ({}^{-}a_{12})^2} \alpha_y \dfrac{\partial f^0}{\partial v}\right]$	$g_2(\boldsymbol{r}, v) = \left[\dfrac{{}^{-}a_{11}\,{}^{-}b_{12} - {}^{-}a_{12}\,{}^{-}b_{11}}{({}^{-}a_{11})^2 + ({}^{-}a_{12})^2} \alpha_x \dfrac{\partial f^0}{\partial v} - \dfrac{{}^{-}a_{11}\,{}^{-}b_{11} + {}^{-}a_{12}\,{}^{-}b_{12}}{({}^{-}a_{11})^2 + ({}^{-}a_{12})^2} \alpha_y \dfrac{\partial f^0}{\partial v}\right]$

$${}^{\omega}a_{11} = \nu[\nu^2 + \omega_C^2 + (\omega_0+\omega_B)^2 - 3\,\omega^2]$$

$${}^{\omega}a_{12} = \omega[3\,\nu^2 + \omega_C^2 + (\omega_0+\omega_B)^2 - \omega^2]$$

$${}^{\omega}b_{11} = 2\,\nu\,\omega$$

$${}^{\omega}b_{12} = -[\nu^2 + (\omega_0+\omega_B)^2 - \omega^2]$$

$${}^{\omega_B}a_{11} = \nu[\nu^2 + \omega_C^2 + (\omega_0+\omega_B)^2 - 3\,\omega_B^2]$$

$${}^{\omega_B}a_{12} = \omega_B[3\,\nu^2 + \omega_C^2 + (\omega_0+\omega_B)^2 - \omega_B^2]$$

$${}^{\omega_B}b_{11} = \frac{\omega_C\,\nu[-(\nu^2+(\omega_0+\omega_B)^2) + 2\,\omega_0\,\omega_B + \omega_B^2]}{\nu^2+\omega_0^2}$$

$${}^{\omega_B}b_{12} = \frac{\omega_C[-\omega_0(\nu^2+(\omega_0+\omega_B)^2) - 2\,\nu^2\,\omega_B + \omega_0\,\omega_B^2]}{\nu^2+\omega_0^2}$$

$${}^{\omega_B}b_{13} = 2\,\nu\,\omega_C\,\omega_0^2$$

$${}^{\omega_B}b_{14} = -\omega_C\,\omega_B(\nu^2 + (\omega_0+\omega_B)^2 - \omega_B^2)$$

$${}^{+}a_{11} = (\omega+\omega_B)[3\,\nu^2 + \omega_B^2 + (\omega_0+\omega_B)^2 - (\omega+\omega_B)^2]$$

$${}^{+}a_{12} = \nu[\nu^2 + \omega_B^2 + (\omega_0+\omega_B)^2 - 3(\omega-\omega_B)^2]$$

$${}^{+}b_{11} = -\frac{\omega_C}{2}\,\frac{\nu(\nu^2+(\omega_0+\omega_B)^2)(\nu^2+\omega^2+\omega_0^2+2\,\omega\,\omega_0) + 2\,\nu(\omega+\omega_B)[\omega(\nu^2+\omega^2-\omega_0^2)+\omega_0(\omega^2-\nu^2-\omega_0^2)] - \nu(\omega+\omega_B)^2(\nu^2+\omega^2+\omega_0^2+2\,\omega\,\omega_0)}{(\nu^2+\omega^2+\omega_0^2)^2+4\,\nu^2\,\omega_0^2}$$

$${}^{+}b_{12} = \frac{\omega_C}{2}\,\frac{-(\nu^2+(\omega_0+\omega_B)^2)[\omega(\nu^2+\omega^2-\omega_0^2)+\omega_0(\omega^2-\nu^2-\omega_0^2)] + 2\,\nu^2(\omega+\omega_B)[2\,\omega\,\omega_0 + (\nu^2+\omega^2+\omega_0^2)] + (\omega+\omega_B)^2[\omega(\nu^2+\omega^2-\omega_0^2)+\omega_0(\omega^2-\nu^2-\omega_0^2)]}{(\nu^2+\omega^2-\omega_0^2)^2+4\,\nu^2\,\omega_0^2}$$

$${}^{-}a_{11} = \nu[\nu^2 + \omega_C^2 + (\omega_0+\omega_B)^2 - 3(\omega-\omega_B)^2]$$

$${}^{-}a_{12} = -(\omega-\omega_B)[3\,\nu^2 + \omega_C^2 + (\omega_0+\omega_B)^2 - (\omega-\omega_B)^2]$$

$${}^{-}b_{11} = \frac{\omega_C}{2}\,\frac{\nu(\nu^2+(\omega_0+\omega_B)^2)[-(\nu^2+\omega^2+\omega_0^2)+2\,\omega\,\omega_0] + 2\,\nu(\omega-\omega_B)[-\omega(\nu^2+\omega^2-\omega_0^2)+\omega_0(\omega^2-\nu^2-\omega_0^2)] - \nu(\omega-\omega_B)^2[-(\nu^2+\omega^2+\omega_0^2)+2\,\omega\,\omega_0]}{(\nu^2+\omega^2-\omega_0^2)^2+4\,\nu^2\,\omega_0^2}$$

$${}^{-}b_{12} = \frac{\omega_C}{2}\,\frac{-(\nu^2+(\omega_0+\omega_B)^2)[\omega(\nu^2+\omega^2-\omega_0^2)-\omega_0(\omega^2-\nu^2-\omega_0^2)] + 2\,\nu^2(\omega-\omega_B)[-2\,\omega\,\omega_0+\nu^2+\omega^2+\omega_0^2] + (\omega-\omega_B)^2[\omega(\nu^2+\omega^2-\omega_0^2)-\omega_0(\omega^2-\nu^2-\omega_0^2)]}{(\nu^2+\omega^2-\omega_0^2)^2+4\,\nu^2\,\omega_0^2}$$

Table 1. Solution f_z^1.

$$\boxed{{}^{\mathrm{inh}}f_x^1(\boldsymbol{r},v,t)} = \underbrace{-\frac{\nu}{\nu^2+\omega_0^2}U_{0x}}_{\langle \mathrm{X}\,1\rangle} \underbrace{-\frac{\omega_0}{\nu^2+\omega_0^2}U_{0y}}_{\langle \mathrm{X}\,2\rangle} \underbrace{-\frac{\nu(\nu^2+\omega^2+\omega_0^2)\cos\omega t+\omega(\nu^2+\omega^2-\omega_0^2)\sin\omega t}{[\nu^2+\omega^2-\omega_0^2]^2+4\nu^2\omega_0^2}\alpha_x\frac{\partial f^0}{\partial v}}_{\langle \mathrm{X}\,3\rangle} \underbrace{+\frac{\omega_0\{(\omega^2-\omega_0^2-\nu^2)\cos\omega t-2\nu\omega\sin\omega t\}}{[\nu^2+\omega^2-\omega_0^2]^2+4\nu^2\omega_0^2}\alpha_y\frac{\partial f^0}{\partial v}}_{\langle \mathrm{X}\,4\rangle}$$

$$\underbrace{-\frac{\omega_{\mathrm{C}}}{\nu(\nu^2+\omega_{\mathrm{C}}^2+(\omega_0+\omega_{\mathrm{B}})^2)}(-\nu\sin\omega_{\mathrm{B}}t+(\omega_0+\omega_{\mathrm{B}})\cos\omega_{\mathrm{B}}t)\,U_{0z}}_{\langle \mathrm{X}\,5\rangle}$$

$$\underbrace{+\frac{c_1(\boldsymbol{r},v)\cdot\omega_{\mathrm{C}}/2}{[\nu^2+\omega^2-(\omega_0+\omega_{\mathrm{B}})^2]^2+4\nu^2(\omega_0+\omega_{\mathrm{B}})^2}}_{\langle \mathrm{X}\,6\rangle}\left[\begin{array}{l}(\nu^2[\omega_0+\omega_{\mathrm{B}}+\omega]+[(\omega_0+\omega_{\mathrm{B}})^2-\omega^2]\,[\omega_0+\omega_{\mathrm{B}}-\omega])\sin\{(\omega+\omega_{\mathrm{B}})\,t\}\\ \qquad +\nu((\omega_0+\omega_{\mathrm{B}})^2+\nu^2+\omega^2-2(\omega_0+\omega_{\mathrm{B}})\,\omega)\cos\{(\omega+\omega_{\mathrm{B}})\,t\}\\ +(\nu^2[\omega_0+\omega_{\mathrm{B}}-\omega]+[(\omega_0+\omega_{\mathrm{B}})^2-\omega^2]\,[\omega_0+\omega_{\mathrm{B}}+\omega])\sin\{(\omega-\omega_{\mathrm{B}})\,t\}\\ \qquad -\nu((\omega_0+\omega_{\mathrm{B}})^2+\nu^2+\omega^2+2(\omega_0+\omega_{\mathrm{B}})\,\omega)\cos\{(\omega-\omega_{\mathrm{B}})\,t\}\end{array}\right]$$

$$+\frac{c_2(\boldsymbol{r},v)\cdot\omega_{\mathrm{C}}/2}{[\nu^2+\omega^2-(\omega_0+\omega_{\mathrm{B}})^2]^2+4\nu^2(\omega_0+\omega_{\mathrm{B}})^2}\left[\begin{array}{l}-\nu((\omega_0+\omega_{\mathrm{B}})^2+\nu^2+\omega^2-2\,\omega(\omega_0+\omega_{\mathrm{B}}))\sin\{(\omega+\omega_{\mathrm{B}})\,t\}\\ \qquad +(\nu^2[\omega_0+\omega_{\mathrm{B}}+\omega]+[(\omega_0+\omega_{\mathrm{B}})^2-\omega^2]\,[\omega_0+\omega_{\mathrm{B}}-\omega])\cos\{(\omega+\omega_{\mathrm{B}})\,t\}\\ +\nu((\omega_0+\omega_{\mathrm{B}})^2+\nu^2+\omega^2+2\,\omega(\omega_0+\omega_{\mathrm{B}}))\sin\{(\omega-\omega_{\mathrm{B}})\,t\}\\ \qquad +(\nu^2[\omega_0+\omega_{\mathrm{B}}-\omega]+[(\omega_0+\omega_{\mathrm{B}})^2-\omega^2]\,[\omega_0+\omega_{\mathrm{B}}-\omega])\cos\{(\omega-\omega_{\mathrm{B}})\,t\}\end{array}\right]$$

$$\underbrace{+\frac{d_1(\boldsymbol{r},v)\cdot\omega_{\mathrm{C}}/2}{[\nu^2+\omega_{\mathrm{B}}^2-(\omega_0+\omega_{\mathrm{B}})^2]^2+4\nu^2(\omega_0+\omega_{\mathrm{B}})^2}}_{\langle \mathrm{X}\,7\rangle}[-\nu(4\,\omega_{\mathrm{B}}^2+4\,\omega_0\,\omega_{\mathrm{B}}+\nu^2+\omega_0^2)+(\nu^2+\omega_0^2)\,(\omega_0+2\,\omega_{\mathrm{B}})\sin\{2\,\omega_{\mathrm{B}}t\}+\nu(\nu^2+\omega_0^2)\cos\{2\,\omega_{\mathrm{B}}t\}]$$

$$+\frac{d_2(\boldsymbol{r},v)\cdot\omega_{\mathrm{C}}/2}{[\nu^2+\omega_{\mathrm{B}}^2-(\omega_0+\omega_{\mathrm{B}})^2]^2+4\nu^2(\omega_0+\omega_{\mathrm{B}})^2}[+\omega_0(4\,\omega_{\mathrm{B}}^2+4\,\omega_0\,\omega_{\mathrm{B}}+\nu^2+\omega_0^2)-\nu(\nu^2+\omega_0^2)\sin\{2\,\omega_{\mathrm{B}}t\}+(\nu^2+\omega_0^2)\,(\omega_0+2\,\omega_{\mathrm{B}})\cos\{2\,\omega_{\mathrm{B}}t\}]$$

$$\underbrace{+\frac{e_1(\boldsymbol{r},v)\cdot\omega_{\mathrm{C}}/2}{[\nu^2+(\omega+\omega_{\mathrm{B}})^2-(\omega_0+\omega_{\mathrm{B}})^2]^2+4\nu^2(\omega_0+\omega_{\mathrm{B}})^2}}_{\langle \mathrm{X}\,8\rangle}\left[\begin{array}{l}([\nu^2+2\,\omega_{\mathrm{B}}^2]\,[\omega_0-\omega]+[\omega_0^2-\omega^2]\,[\omega+\omega_0+2\,\omega_{\mathrm{B}}])\sin\omega t-\nu(\nu^2+(\omega_0+\omega_{\mathrm{B}})^2\\ \quad +(\omega+\omega_{\mathrm{B}})^2+2(\omega_0+\omega_{\mathrm{B}})\,(\omega+\omega_{\mathrm{B}}))\cos\omega t+\nu^2[\omega+\omega_0+2\,\omega_{\mathrm{B}}]+(\omega-\omega_0)\,((\omega+\omega_{\mathrm{B}})^2\\ \quad -(\omega_0+\omega_{\mathrm{B}})^2)\sin\{(\omega+2\,\omega_{\mathrm{B}})\,t\}+\nu(\nu^2+(\omega-\omega_0)^2)\cos\{(\omega+2\,\omega_{\mathrm{B}})\,t\}\end{array}\right]$$

$$+\frac{e_2(\boldsymbol{r},v)\cdot\omega_{\mathrm{C}}/2}{[\nu^2+(\omega+\omega_{\mathrm{B}})^2-(\omega_0+\omega_{\mathrm{B}})^2]^2+4\nu^2(\omega_0+\omega_{\mathrm{B}})^2}\left[\begin{array}{l}(-\nu[\nu^2+(\omega+\omega_{\mathrm{B}})^2-(\omega_0+\omega_{\mathrm{B}})^2]+2\,\nu(\omega_0+\omega_{\mathrm{B}})\;[\omega+\omega_0+2\,\omega_{\mathrm{B}}])\sin\omega t+(-\nu^2(\omega-\omega_0)\\ \quad +[(\omega_0+\omega_{\mathrm{B}})^2-(\omega+\omega_{\mathrm{B}})^2]\,[\omega+\omega_0+2\,\omega_{\mathrm{B}}])\cos\omega t+(-\nu[\nu^2+(\omega+\omega_{\mathrm{B}})^2-(\omega_0+\omega_{\mathrm{B}})^2]\\ \quad +2\,\nu(\omega_0+\omega_{\mathrm{B}})\;(\omega-\omega_0))\sin\{(\omega+2\,\omega_{\mathrm{B}})\,t\}+(2\,\nu^2(\omega_0+\omega_{\mathrm{B}})+[\nu^2+(\omega+\omega_{\mathrm{B}})^2\\ \quad -(\omega_0+\omega_{\mathrm{B}})^2]\;(\omega-\omega_0))\cos\{(\omega+2\,\omega_{\mathrm{B}})\,t\}\end{array}\right]$$

$$\underbrace{+\frac{g_1(\boldsymbol{r},v)\cdot\omega_{\mathrm{C}}/2}{[\nu^2+(\omega-\omega_{\mathrm{B}})^2-(\omega_0+\omega_{\mathrm{B}})^2]^2+4\nu^2(\omega_0+\omega_{\mathrm{B}})^2}}_{\langle \mathrm{X}\,9\rangle}\left[\begin{array}{l}(2\,\nu^2(\omega_0+\omega_{\mathrm{B}})+(\omega-\omega_0)\,[\nu^2+(\omega-\omega_{\mathrm{B}})^2-(\omega_0+\omega_{\mathrm{B}})^2])\sin\omega t+\nu(\nu^2+(\omega-\omega_{\mathrm{B}})^2\\ -(\omega_0+\omega_{\mathrm{B}})^2+2(\omega_0+\omega_{\mathrm{B}})\,[\omega_0+2\,\omega_{\mathrm{B}}-\omega])\cos\omega t+(2\,\nu^2(\omega_0+\omega_{\mathrm{B}})-(\omega+\omega_0)\,[\nu^2+(\omega-\omega_{\mathrm{B}})^2\\ -(\omega+\omega_{\mathrm{B}})^2])\sin\{(\omega-2\,\omega_{\mathrm{B}})\,t\}-\nu(\nu^2+(\omega-\omega_{\mathrm{B}})^2\\ -(\omega_0+\omega_{\mathrm{B}})^2+2(\omega_0+\omega_{\mathrm{B}})\,(\omega+\omega_0))\cos\{(\omega-2\,\omega_{\mathrm{B}})\,t\}\end{array}\right]$$

$$+\frac{g_2(\boldsymbol{r},v)\cdot\omega_{\mathrm{C}}/2}{[\nu^2+(\omega-\omega_{\mathrm{B}})^2-(\omega_0+\omega_{\mathrm{B}})^2]^2+4\nu^2(\omega_0+\omega_{\mathrm{B}})^2}\left[\begin{array}{l}(2\,\nu(\omega_0+\omega_{\mathrm{B}})\,[\omega-\omega_0-2\,\omega_{\mathrm{B}}]-\nu[\nu^2+(\omega-\omega_{\mathrm{B}})^2-(\omega_0+\omega_{\mathrm{B}})^2])\sin\omega t+(2\,\nu^2(\omega_0+\omega_{\mathrm{B}})\\ +[\nu^2+(\omega-\omega_{\mathrm{B}})^2-(\omega_0+\omega_{\mathrm{B}})^2]\;[\omega-\omega_0-2\,\omega_{\mathrm{B}}]\cos\omega t\\ +(2\,\nu(\omega_0+\omega_{\mathrm{B}})\,(\omega+\omega_0)+\nu[\nu^2+(\omega-\omega_{\mathrm{B}})^2-(\omega_0+\omega_{\mathrm{B}})^2])\sin\{(\omega-2\,\omega_{\mathrm{B}})\,t\}\\ +(2\,\nu^2(\omega_0+\omega_{\mathrm{B}})-(\omega+\omega_0)\;[\nu^2+(\omega-\omega_{\mathrm{B}})^2-(\omega_0+\omega_{\mathrm{B}})^2]\cos\{(\omega-2\,\omega_{\mathrm{B}})\,t\}\end{array}\right]$$

Table 2. Solution f_x^1.

$$
\boxed{{}^{\text{inh}}f_y^1(\boldsymbol{r}, v, t)} = \underbrace{\frac{\omega_0}{\nu^2+\omega_0^2}\,U_{0x}}_{\langle \text{Y}\,1\rangle} - \underbrace{\frac{\nu}{\nu^2+\omega_0^2}\,U_{0y}}_{\langle \text{Y}\,2\rangle} - \underbrace{\frac{\omega_0\{(\omega^2-\omega_0^2-\nu^2)\cos\omega t - 2\,\nu\,\omega\sin\omega t\}}{[\nu^2+\omega^2-\omega_0^2]^2+4\,\nu^2\,\omega_0^2}\,\alpha_x\,\frac{\partial f^0}{\partial v}}_{\langle \text{Y}\,3\rangle} - \underbrace{\frac{\nu(\nu^2+\omega^2+\omega_0^2)\cos\omega t + \omega(\nu^2+\omega^2-\omega_0^2)\sin\omega t}{[\nu^2+\omega^2-\omega_0^2]^2+4\,\nu^2\,\omega_0^2}\,\alpha_y\,\frac{\partial f^0}{\partial v}}_{\langle \text{Y}\,4\rangle}
$$

$$
\underbrace{- \frac{\omega_\text{C}}{\nu(\nu^2+\omega_\text{C}^2+(\omega_0+\omega_\text{B})^2)}\,((\omega_0+\omega_\text{B})\sin\omega_\text{B} t + \nu\cos\omega_\text{B} t)\,U_{0z}}_{\langle \text{Y}\,5\rangle}
$$

$$
\underbrace{+ \frac{c_1(\boldsymbol{r}, v)\cdot\omega_\text{C}/2}{[\nu^2+\omega^2-(\omega_0+\omega_\text{B})^2]^2+4\,\nu^2(\omega_0+\omega_\text{B})^2}}_{\langle \text{Y}\,6\rangle}
\begin{bmatrix}
\nu((\omega_0+\omega_\text{B})^2+\nu^2+\omega^2-2\,\omega(\omega_0+\omega_\text{B}))\sin\{(\omega+\omega_\text{B})\,t\} + \nu((\omega_0+\omega_\text{B})^2+\nu^2+\omega^2+2\,\omega(\omega_0+\omega_\text{B})) \\
\sin\{(\omega-\omega_\text{B})\,t\} - (\nu^2[\omega_0+\omega_\text{B}+\omega]+[(\omega_0+\omega_\text{B})^2-\omega^2]\,[\omega_0+\omega_\text{B}-\omega])\cos\{(\omega+\omega_\text{B})\,t\} \\
+ (\nu^2[\omega_0+\omega_\text{B}+\omega]+[(\omega_0+\omega_\text{B})^2-\omega^2]\,[\omega_0+\omega_\text{B}-\omega])\cos\{(\omega-\omega_\text{B})\,t\}
\end{bmatrix}
$$

$$
+ \frac{c_2(\boldsymbol{r}, v)\cdot\omega_\text{C}/2}{[\nu^2+\omega^2-(\omega_0+\omega_\text{B})^2]^2+4\,\nu^2(\omega_0+\omega_\text{B})^2}
\begin{bmatrix}
(\nu^2(\omega_0+\omega_\text{B}+\omega)+[(\omega_0+\omega_\text{B})^2-\omega^2]\,[\omega_0+\omega_\text{B}-\omega])\sin\{(\omega+\omega_\text{B})\,t\} \\
- (\nu^2[\omega_0+\omega_\text{B}-\omega]+[(\omega_0+\omega_\text{B})^2-\omega^2]\,[\omega_0+\omega_\text{B}+\omega])\sin\{(\omega-\omega_\text{B})\,t\} \\
+ \nu((\omega_0+\omega_\text{B})^2+\nu^2+\omega^2-2\,\omega(\omega_0+\omega_\text{B}))\cos\{(\omega+\omega_\text{B})\,t\} \\
+ \nu((\omega_0+\omega_\text{B})^2+\nu^2+\omega^2+2\,\omega(\omega_0+\omega_\text{B}))\cos\{(\omega-\omega_\text{B})\,t\}
\end{bmatrix}
$$

$$
\underbrace{+ \frac{d_1(\boldsymbol{r}, v)\cdot\omega_\text{C}/2}{[\nu^2+\omega_\text{B}^2-(\omega_0+\omega_\text{B})^2]^2+4\,\nu^2(\omega_0+\omega_\text{B})^2}}_{\langle \text{Y}\,7\rangle}\,[\omega_0(4\,\omega_\text{B}^2+4\,\omega_0\,\omega_\text{B}+\nu^2+\omega_0^2)+\nu(\nu^2+\omega_0^2)\sin\{2\,\omega_\text{B} t\}+(\nu^2+\omega_0^2)(\omega_0+2\,\omega_\text{B})\cos\{2\,\omega_\text{B} t\}]
$$

$$
+ \frac{d_2(\boldsymbol{r}, v)\cdot\omega_\text{C}/2}{[\nu^2+\omega_\text{B}^2-(\omega_0+\omega_\text{B})^2]^2+4\,\nu^2(\omega_0+\omega_\text{B})^2}\,[\nu(4\,\omega_\text{B}^2+4\,\omega_0\,\omega_\text{B}+\nu^2+\omega_0^2)+(\nu^2+\omega_0^2)(\omega_0+2\,\omega_\text{B})\sin\{2\,\omega_\text{B} t\}+\nu(\nu^2+\omega_0^2)\cos\{2\,\omega_\text{B} t\}]
$$

$$
\underbrace{+ \frac{e_1(\boldsymbol{r}, v)\cdot\omega_\text{C}/2}{[\nu^2+(\omega+\omega_\text{B})^2-(\omega_0+\omega_\text{B})^2]^2+4\,\nu^2(\omega_0+\omega_\text{B})^2}}_{\langle \text{Y}\,8\rangle}
\begin{bmatrix}
\nu(\nu^2+(\omega_0+\omega_\text{B})^2+(\omega+\omega_\text{B})^2+2\,\nu(\omega_0+\omega_\text{B})\,(\omega+\omega_\text{B}))\sin\omega t \\
+ (-(\omega+\omega_\text{B})[\nu^2+(\omega+\omega_\text{B})^2-(\omega_0+\omega_\text{B})^2] + (\omega_0+\omega_\text{B})[\nu^2+(\omega_0+\omega_\text{B})^2-(\omega+\omega_\text{B})^2])\cos\omega t \\
+ \nu(\nu^2+(\omega_0+\omega_\text{B})^2+(\omega+\omega_\text{B})^2-2(\omega_0+\omega_\text{B})\,(\omega+\omega_\text{B}))\sin\{(\omega+2\,\omega_\text{B})\,t\} \\
+ (-2\,\nu^2(\omega_0+\omega_\text{B})-(\omega-\omega_0)\,[\nu^2+(\omega+\omega_\text{B})^2-(\omega_0+\omega_\text{B})^2])\cos\{(\omega+2\,\omega_\text{B})\,t\}
\end{bmatrix}
$$

$$
+ \frac{e_2(\boldsymbol{r}, v)\cdot\omega_\text{C}/2}{[\nu^2+(\omega+\omega_\text{B})^2-(\omega_0+\omega_\text{B})^2]^2+4\,\nu^2(\omega_0+\omega_\text{B})^2}
\begin{bmatrix}
(-2\,\nu^2(\omega_0+\omega_\text{B})+[\omega+\omega_0+2\,\omega_\text{B}][\nu^2+(\omega+\omega_\text{B})^2-(\omega_0+\omega_\text{B})^2])\sin\omega t \\
+ \nu(\nu^2+(\omega+\omega_\text{B})^2-(\omega_0+\omega_\text{B})^2+2(\omega_0+\omega_\text{B})\,[\omega+\omega_0+2\,\omega_\text{B}])\cos\omega t \\
+ (2\,\nu^2(\omega_0+\omega_\text{B})+(\omega-\omega_0)[\nu^2+(\omega+\omega_\text{B})^2-(\omega_0+\omega_\text{B})^2])\sin\{(\omega+2\,\omega_\text{B})\,t\} \\
+ (\nu[\nu^2+(\omega+\omega_\text{B})^2-(\omega_0+\omega_\text{B})^2]-2\,\nu(\omega-\omega_0)\,(\omega_0+\omega_\text{B}))\cos\{(\omega+2\,\omega_\text{B})\,t\}
\end{bmatrix}
$$

$$
\underbrace{+ \frac{g_1(\boldsymbol{r}, v)\cdot\omega_\text{C}/2}{[\nu^2+(\omega-\omega_\text{B})^2-(\omega_0+\omega_\text{B})^2]^2+4\,\nu^2(\omega_0+\omega_\text{B})^2}}_{\langle \text{Y}\,9\rangle}
\begin{bmatrix}
(2\,\nu(\omega_0+\omega_\text{B})[\omega_0+2\,\omega_\text{B}-\omega]+\nu[\nu^2+(\omega-\omega_\text{B})^2-(\omega_0+\omega_\text{B})^2)\sin\omega t \\
+ (-2\,\nu^2(\omega_0+\omega_\text{B})[\omega_0+2\,\omega_\text{B}-\omega]\,[\nu^2+(\omega-\omega_\text{B})^2-(\omega_0+\omega_\text{B})^2])\cos\omega t \\
+ (2\,\nu(\omega_0+\omega_\text{B})[\omega_0+2\,\omega_\text{B}-\omega]+\nu[\nu^2+(\omega-\omega_\text{B})^2-(\omega_0+\omega_\text{B})^2])\sin\{(\omega-2\,\omega_\text{B})\,t\} \\
+ (2\,\nu^2(\omega_0+\omega_\text{B})-(\omega+\omega_0)\,[\nu^2+(\omega-\omega_\text{B})^2-(\omega_0-\omega_\text{B})^2])\cos\{(\omega-2\,\omega_\text{B})\,t\}
\end{bmatrix}
$$

$$
+ \frac{g_2(\boldsymbol{r}, v)\cdot\omega_\text{C}/2}{[\nu^2+(\omega-\omega_\text{B})^2-(\omega_0+\omega_\text{B})^2]^2+4\,\nu^2(\omega_0+\omega_\text{B})^2}
\begin{bmatrix}
(2\,\nu^2(\omega_0+\omega_\text{B})+[\nu^2+(\omega-\omega_\text{B})^2-(\omega_0+\omega_\text{B})^2]\,[\omega-\omega_0-2\,\omega_\text{B}])\sin\omega t \\
+ (\nu[\nu^2+(\omega-\omega_\text{B})^2-(\omega_0+\omega_\text{B})^2]-2\,\nu(\omega-\omega_0)\,(\omega_0+\omega_\text{B}))\cos\omega t \\
+ (-2\,\nu^2(\omega_0+\omega_\text{B})+(\omega+\omega_0)[\nu^2+(\omega-\omega_\text{B})^2-(\omega_0+\omega_\text{B})^2])\sin\{(\omega-2\,\omega_\text{B})\,t\} \\
+ (\nu[\nu^2+(\omega-\omega_\text{B})^2-(\omega_0+\omega_\text{B})^2]+2\,\nu(\omega+\omega_0)\,(\omega_0+\omega_\text{B}))\cos\{(\omega-2\,\omega_\text{B})\,t\}
\end{bmatrix}
$$

Table 3. Solution $f_y{}^1$.

The final form of

$$^{\text{inh}}w(\mathbf{r}, v, t; f^0) = {}^{\text{inh}}f^1_x(\mathbf{r}, v, t; f^0) + i\, {}^{\text{inh}}f^1_y(\mathbf{r}, v, t; f^0) \tag{24}$$

can be got by calculating the time integrals; see Tables 2 and 3.

Knowing the solution vector $^{\text{inh}}\mathbf{f}^1(\mathbf{r}, v, t; f^0)$ one can discuss the physical meaning of the several terms of $^{\text{inh}}\mathbf{f}^1$.

5. Physical Meaning of the Solution

For the purpose of simplifying the discussion some arrangements are made, permitting a more systematic treatment.

- Firstly each term shall be characterized with respect to its kind of time dependence (abbreviation K: "kind").
- The presence of each term originates in a physical mechanism, which shall be given next (abbreviation O: "origin").
- Each term of the solution $^{\text{inh}}\mathbf{f}^1$ has its influence on the current density distribution $\mathbf{j}$ according to the relation

$$\mathbf{j}(\mathbf{r}, t) = q \int d\mathbf{v}\, \mathbf{v}\, f(\mathbf{r}, \mathbf{v}, t) = \tfrac{4}{3}\, q \int_0^\infty dv\, v^3\, \mathbf{f}^1(\mathbf{r}, v, t) \tag{25}$$

(abbreviation C: "consequence").

If necessary the statements K, O, and C will be completed by annotations (abbreviation A).

It should be pointed out that conclusions from the solution $\mathbf{f}^1(f^0)$ to thet temporal behaviour of the current density only can be drawn if the isotropic part f^0 is not time-dependent.

Explanation of the terms of $^{\text{inh}}f^1_z$ (compare Table 1).

⟨Z 1⟩

K: This term is not-time-dependent.

O: The term is caused by the constant electric field and the spatial gradient of the distribution function in direction of the z axis. ⟨Z 1⟩ is dependent on the parameters ω_0, ω_C, ω_B of the magnetic fields.

C: The result is a time-constant drift current in direction of the z axis.

⟨Z 2⟩

K: These parts of the solution oscillate with the fundamental frequency ω of the alternating electric field.

O: They are caused by the z component of the alternating electric field. ⟨Z 2⟩ is influenced by the parameters ω_0, ω_C, ω_B of the magnetic fields.

C: This results in an alternating current (in z direction) oscillating with the fundamental frequency ω of the electric a.c. field.

A: In the limit $\omega \to 0$, $\alpha_z \to \alpha_{0z}$ the term ⟨Z 2⟩ reduces to the term ⟨Z 1⟩ if only homogeneous plasmas are under consideration.

⟨Z 3⟩

K: These parts oscillate with the fundamental rotation frequency ω_B of the circularly polarized magnetic field.

O: 1) Terms $\delta_1 \sin \omega_B t$ and $\delta_2 \cos \omega_B t$. The *simultaneous* presence of the driving forces $\boldsymbol{U}_{0x}$ and $\boldsymbol{U}_{0y}$ [compare Eq. (21)] *and* the rotating magnetic field originates an $\boldsymbol{E} \times \boldsymbol{B}$ drift (cross drift) [2] in direction of the z axis. If the quantities $\boldsymbol{U}_{0x}$ and $\boldsymbol{U}_{0y}$ disappear or the quantity ω_C is equal to zero (that is tantamount to the fact, that one of the both reasons necessary to the development of a cross drift mechanism disappears) the terms $\delta_1 \sin \omega_B t$ and $\delta_2 \cos \omega_B t$ are zero, too.

2) Terms $\Delta_1 \sin \omega_B t$ and $\Delta_2 \cos \omega_B t$. The *time-dependent* circularly polarized magnetic field produces an alternating electric field according to Faraday's induction law. This a.c. field is the origin for a particle drift. A disappearing rotating field ($\omega_C = 0$) involves the disappearance of the induced electric field, i. e.

$$\Delta_1 = \Delta_2 = 0 .$$

C: The result is an alternating current oscillating with the fundamental frequency ω_B in z direction. This current is a cross drift current in case 1), a simple drift current in case 2).

⟨Z 4⟩ [⟨Z 5⟩]

K: These parts oscillate with the sum [the difference] of the fundamental frequency ω and the fundamental rotation frequency ω_B.

O: The *simultaneous* presence of the "driving forces" $\boldsymbol{U}_x$ and $\boldsymbol{U}_y$ [compare Eq. (21)] *and* the rotating magnetic field originates a cross drift in the direction of the z axis. The disappearance of one of these causes (or both of them) makes the terms $\langle Z\,4\rangle$ and $\langle Z\,5\rangle$ vanishing.

C: There will appear a time-periodic cross drift term in z direction with the frequency $\omega+\omega_B$ $[\omega-\omega_B]$.

It is now necessary to discuss the parts of ${}^{\text{inh}}f_x^1$, (the terms of ${}^{\text{inh}}f_y^1$ can be discussed in a similary way).

Explanation of the terms of ${}^{\text{inh}}f_x^1$ (compare Table 2).

$\langle X\,1\rangle$

K: This part is time-independent.

O: The term is produced by the driving force $\boldsymbol{U}_{0x}$. $\langle X\,1\rangle$ is influenced by the magnetic field $\boldsymbol{B}_0$ perpendicular to $\boldsymbol{U}_{0x}$ [compare factor $(\nu^2+\omega_0{}^2)^{-1}$].

C: The arising drift current in direction of the x axis is time-independent.

$\langle X\,2\rangle$

K: The term $\langle X\,2\rangle$ is not time-dependent.

O: The *simultaneous* presence of the force $\boldsymbol{U}_{0y}$ *and* the constant magnetic field $\boldsymbol{B}_0$ perpendicular to $\boldsymbol{U}_{0y}$ yields a cross drift in x direction.

C: The cross drift current appearing in x direction is constant with respect to time.

$\langle X\,3\rangle$

K: These parts oscillate with the fundamental frequency ω of the alternating electric field.

O: This term is caused by the x component $\boldsymbol{E}_x$ of the alternating electric field. $\langle X\,3\rangle$ is influenced by the constant magnetic field being perpendicular to $\boldsymbol{E}_x$ [compare the factor:

$$([\nu^2+\omega^2-\omega_0{}^2]^2+4\,\nu^2\,\omega_0{}^2)^{-1}$$

in $\langle X\,3\rangle$].

C: This results in a drift current oscillating with the fundamental frequency ω in x direction.

$\langle X\,4\rangle$

K: These parts oscillate with the frequency ω of the alternating electric field.

O: The simultaneous presence of $\boldsymbol{U}_y$ and $\boldsymbol{B}_0$ being perpendicular to $\boldsymbol{U}_y$ gives rise to a cross drift in x direction.

C: The arising cross drift current oscillates with a frequency ω in x direction.

$\langle X\,5\rangle$

K: These parts oscillate with the fundamental frequency ω_B of the rotating magnetic field.

O: The simultaneous presence of $\boldsymbol{U}_{0z}$ and the circularly polarized magnetic field being perpendicular to $\boldsymbol{U}_{0z}$ leads to a cross drift in x direction.

C: The alternating cross drift current oscillates with the frequency ω_B in x direction.

$\langle X\,6\rangle$

K: These terms involve oscillations with frequencies $\omega\pm\omega_B$.

O: The simultaneous presence of the driving force $\boldsymbol{U}_z$ and the circularly polarized magnetic field gives rise to a cross drift in direction of the x axis.

C: The arising alternating cross drift currents in x direction have frequencies $\omega\pm\omega_B$.

$\langle X\,7\rangle$

K: These parts are either time-independent or periodical in time with the frequency $2\,\omega_B$.

O: The existence of these terms has two origins:

O1: The forces in $\langle Z\,3\rangle$ characterized by δ_1 and δ_2 yield in connection with the rotating magnetic field a cross drift in x direction.

O2: The simultaneous presence of the induced electric field $\boldsymbol{E}_i^R$ (given by Δ_1 and Δ_2 in $\langle Z\,3\rangle$) and the circularly polarized magnetic field rotating perpendicular to $\boldsymbol{E}_i^R$ give rise to a cross drift in direction of the x axis, too.

C: This will lead to cross drift currents along the x axis which are either time-independent or oscillating with the frequency $2\,\omega_B$.

Explanations to $\langle X\,7\rangle$	First influence of the rotating magnetic field	Second influence of the rotating magnetic field
0 1 :	The driving forces U_{0x} and U_{0y} cause in connection with the rotating magnetic field a cross drift in z direction.	The force which drives the electrons in z direction according to the first influence of the rotating magnetic field causes moreover in connection with the circularly polarized magnetic field a cross drift in x direction.
0 2 :	The time-dependent rotating magnetic field induces an electric field in z direction.	The induced electric field and the polarized magnetic field rotating perpendicular to it give rise to a cross dirft in x direction.

Table 4.

A: In this connection it should be emphasized that the rotating field takes part in the development of a cross drift mechanism in a twofold manner:

The physical situation described in Table 4 can be proved in some mathematical respect by investigating the terms of ⟨X 7⟩:

All the frequency functions ${}^{\omega_B}b_i$ $(i=1\ldots4)$ are homogeneous functions of first degree with respect to ω_C. Therefore the quantities d_1 and d_2 are proportional to ω_c^2 (compare Table 1; the additional bearing of ω_c^2 in ${}^{\omega_B}a_{11}$ and ${}^{\omega_B}a_{12}$ is not important here and can be ignored). Another mathematical hint for the twofold influence of the rotating field is the fact that the products from $\sin\omega_B t$ and $\cos\omega_B t$ – which must be calculated according to Eq. (23) – yield constant parts and terms with $2\,\omega_B$ in agreement with ⟨X 7⟩.

⟨X 8⟩ [⟨X 9⟩]

K: These parts are periodical in time with frequencies ω or $\omega+2\,\omega_B$ [ω or $\omega-2\,\omega_B$].

O: This case is completely analogous to that of ⟨X 7⟩. This can be easily seen if one replaces the forces $\boldsymbol{U}_{0x}$ and $\boldsymbol{U}_{0y}$ in ⟨X 7⟩ by $\boldsymbol{U}_x$ and $\boldsymbol{U}_y$. The forces in ⟨Z 4⟩ and ⟨Z 5⟩ characterized by e_1, e_2 and g_1, g_2 yield in connection with the rotating magnetic field a cross drift in x direction.

C: The mechanism causes cross drift currents oscillating with frequencies ω or $\omega+2\,\omega_B$ [ω or $\omega-2\,\omega_B$]. The proof that the circularly polarized magnetic field is engaged in the cross drift processes in a twofold manner can be given similar to ⟨X 7⟩.

Overlooking the terms of ${}^{\text{inh}}f_x^1$ it can be stated that the terms from ⟨X 1⟩ to ⟨X 4⟩ are caused by the constant fields (constant electric field, density- and temperature gradients), the alternating electric field, and by the cross drift mechanism produced by these constant fields and the constant magnetic field. On the other hand the terms from ⟨X 5⟩ and ⟨X 9⟩ exclusively give rise to cross drift processes in which the rotating magnetic field plays an important role (⟨X 5⟩ – ⟨X 9⟩ disappear if the circularly polarized magnetic field is not present).

In Table 5 all terms of $\boldsymbol{f}^1$ are listed with respect to their time behaviour.

6. Conclusion

In the last section it was shown that higher current harmonics are produced by a cross drift mechanism if a weakly ionized plasma is under the action of an alternating electric field and a circularly polarized magnetic field. It is worth noting that the higher harmonics disappear when one replaces the time-dependent rotating magnetic field by a constant magnetic field by putting the rotation frequency $\omega_B=0$ in the solution ${}^{\text{inh}}\boldsymbol{f}^1(\boldsymbol{r},v,t)$. Further, it should be mentioned that higher harmonic terms with a frequency of $2\,\omega_B$ are present in ${}^{\text{inh}}\boldsymbol{f}^1$ if only a *constant electric* field and a rotating magnetic field are taken into account. All these facts show that the time-dependent circularly polarized magnetic field (in connection with its induced electric field) plays the decisive role for the generation of "higher magnetic cross harmonics".

Finally we mention the important condition of time-independence of the isotropic part f^0 used in calculating the solution vector $\boldsymbol{f}^1(\boldsymbol{r},v,t;f^0)$. In 1948 MARGENAU and HARTMANN [18] found that higher

f^1	Time-Independent Terms	Time-Dependent Terms						
		ω	ω_B	$2\omega_B$	$\omega+\omega_B$	$\omega-\omega_B$	$\omega+2\omega_B$	$\omega-2\omega_B$
f_x^1	⟨X 1⟩ ⟨X 2⟩ ⟨X 7⟩	⟨X 3⟩ ⟨X 4⟩ ⟨X 8⟩ ⟨X 9⟩	⟨X 5⟩	⟨X 7⟩	⟨X 6⟩	⟨X 6⟩	⟨X 8⟩	⟨X 9⟩
f_y^1	⟨Y 1⟩ ⟨Y 2⟩ ⟨Y 7⟩	⟨Y 3⟩ ⟨Y 4⟩ ⟨Y 8⟩ ⟨Y 9⟩	⟨Y 5⟩	⟨Y 7⟩	⟨Y 6⟩	⟨Y 6⟩	⟨Y 8⟩	⟨Y 9⟩
f_z^1	⟨Z 1⟩	⟨Z 2⟩	⟨Z 3⟩		⟨Z 4⟩	⟨Z 5⟩		

Table 5.

[18] H. MARGENAU and L. M. HARTMAN, Phys. Rev. **73**, 309 [1948].

current harmonics appear in plasmas subjected to an a.c. field. These authors made the following ansatz for the isotropic part f^0:

$$f(v,t) = f_0^0(v) + f_2^0(v)\exp(2\,i\,\omega\,t) + v_x[f_1^1(v)\exp(i\,\omega\,t) + f_3^1(v)\exp(3\,i\,\omega\,t)].$$

In recent time similar series expansions hav been applied in order to calculate the amplitudes of higher harmonics [19–22] even if a *constant* magnetic field is taken into account [23, 24]. The essential result of all these papers following the treatment of Margenau and Hartmann can be derived as follows:

Suppose the isotropic part f^0 to be time-dependent. By expanding f^0 into a Fourier series it is possible to obtain an infinite number of current harmonics in general. If in addition a constant magnetic field is taken into account this will modify the amplitudes of the higher harmonics without changing the time-behaviour, however, which is exclusively determined by the alternating electric field itself.

On the contrary the field configuration investigated in this paper (a.c. and d.c. field, rotating magnetic field, induced electric field) gives rise to a finite number of "higher magnetic cross harmonics" in the plasma under the condition that f^0 is time-independent. In another paper [17] it is planned to give an explicit calculation of the isotropic part f^0 for the field configuration here discussed making use of the additional assumption that certain relations of homogeneity [14, 15] are valid.

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Generalized Electron Cyclotron Resonance in Weakly Ionized Time-Varying Magnetoplasmas

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The electron distribution function is calculated explicitly for a weakly ionized plasma under the action of an alternating electric field $\boldsymbol{E}=\{0, 0, E_{0z}\cos\omega t\}$ and a circularly polarized magnetic field $\boldsymbol{B}^{\mathrm{R}}=B_{\mathrm{c}}\{\cos\omega_{\mathrm{B}} t, \sin\omega_{\mathrm{B}} t, 0\}$ rotating perpendicular to the a. c. field. Furthermore, a constant magnetic field $\boldsymbol{B}_0=\{0, 0, B_0\}$ is taken into account. The isotropic part f^0 of the electron distribution function which contains, in special cases, well-known standard distributions (distributions of DRUYVENSTEYN, DAVYDOV, MARGENAU, ALLIS, FAIN, GUREVIČ) shows a resonance behaviour if the frequencies ω, $\omega_{\mathrm{c}}=(q/m)\,B_{\mathrm{c}}$, $\omega_0=(q/m)\,B_0$, and ω_{B} satisfy the relation

$$\omega=\sqrt{\omega_{\mathrm{c}}^2+(\omega_0+\omega_{\mathrm{B}})^2}.$$

This can be understood as a generalized cyclotron resonance phenomenon.

A general treatment of weakly ionized time-varying magnetoplasmas by means of the kinetic theory (Boltzmann equation) was given in the paper [1]. The application to a special field configuration consisting of an alternating electric field and a circularly polarized magnetic field was described in a following paper [2]. Assuming the isotropic part f^0 of the electron distribution function to be time-independent one can determine the direction-dependent part $\mathbf{f}^1$ in terms of the unknown part f^0. This leads to higher harmonic terms in the solution $\mathbf{f}^1\{f^0\}$ which can be explained by a cross drift mechanism. It is worth noting that an explanation of these terms does not require the explicit knowledge of the quantity f^0.

In this paper we are interested in a precise determination of f^0 in order to calculate $\mathbf{f}^1$ (and also f as a whole) in its explicit form. This will enable us to investigate a special cyclotron resonance phenomenon appearing when the characteristic frequencies of the external fields satisfy a simple relation.

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